

## Optimization of Fuzzy Matrix Games of Order 4 X 3

R. Senthil Kumar, S. Kumaraghuru

CMS College of Science &Commerce, Coimbatore  
Chikkanna Government Arts College, Tirupur

### Abstract

In this paper, we consider a solution for Fuzzy matrix game with fuzzy pay offs. The Solution of Fuzzy matrix games with pure strategies with maximin – minimax principle is discussed. A method takes advantage of the relationship between fuzzy sets and fuzzy matrix game theories can be offered for multicriteria decision making. Here,  $m \times n$  pay off matrix is reduced to  $4 \times 3$  pay off matrix.

**Key words:** strategy, fuzzy set, fuzzy payoffs, fuzzy matrix games.

### I. Introduction

The problem of Game theory is defined as a body of knowledge that deals with making decisions when two or more intelligent and rational opponents are involved under conditions of conflict and competitions. In practical life, it is required to take decisions in a competing situation when there are two or more opposite teams with conflicting interests and the outcome is controlled by the decisions of all parties concerned. Problems of engineering, industry and other areas are taking on dimensions that no longer allow satisfactory solution by currently employed methods. These are complex and interrelated problems the solution of which depends on the goals pursued by different interested parties. It should be noted that it is often quite difficult to obtain precise information for practical applications. In many cases, it is only possible to get rough values. Since precise information is required, the lack of accuracy will affect the quality of the solution obtained. In the following discussion, the theory of matrix games is used alongside the theory of fuzzy sets, which offers the possibility to take into account the phenomenon known as fuzziness. A solution of two person zero sum game is offered for a matrix with fuzzy payoffs. The suggested approach draws on a commonly used method for solving a classic game and is referred to as fuzzy linear programming. To solve fuzzy linear programming problems, some additional well known models aimed to assess fuzzy numbers are also offered. To solve two person zero sum game with fuzzy payoff, the criterion of minimax is used. Three kinds of concepts of minimax equilibrium strategies are defined and their properties should be investigated. It is shown that the equilibrium strategies considered may be characterized as Nash equilibrium strategies. The properties of values of fuzzy matrix games are investigated by means of possibility and necessity measures.

### II. Classical Matrix Games

In this section matrix games are described with respect to two nonempty sets  $S_1$  and  $S_2$ , the strategy sets of players I and II and gain function  $A(S_1, S_2)$  defined for the Cartesian product  $S_1 \times S_2$ . Use is generally made of the symbolic notation  $\Omega = (S_1, S_2, A)$ . For a solution, the two players orientate themselves with respect to the payoffs bound, namely  $a(S_1) = \min a(s_1, s_2)$  warranty bound for  $s_1 \in S_1$ .

$a(S_2) = \max a(s_1, s_2)$  warranty bound for  $s_2 \in S_2$ .

Choosing the optimum warranty bounds, we obtain:

$a(S_1, S_2) = \max a(s_1, s_2)$

and  $a(S_1, S_2) = \min a(s_1, s_2)$ .

To arrive at a solution, it is necessary to rely on equilibrium that manifests itself as a saddle point. For the game  $\Omega = (S_1, S_2, A)$  a saddle point will be obtained, if and only in the expressions  $\max a(s_1, s_2)$  and  $\min a(s_1, s_2)$  exist and are equal, (ie) if  $\max a(s_1, s_2) = \min a(s_1, s_2)$ .

Since the strategy sets are finite, the expressions do always exist. The equilibrium strategies of player 1 are those strategies  $s_1 \in S_1$ , for which the  $s_2$  infimum reaches the maximum relative to  $s_1$ . Similarly, the equilibrium strategies for player 2 are those strategies  $s_2 \in S_2$ , for which the  $s_2$  supremum reaches the minimum relative to  $s_2$ . The criterion

$\max \min a(s_1, s_2) = \min \max a(s_1, s_2) = v$  is a well known min- max principle with the value  $v$  being the game value.

### III. Fuzzy Matrix Games

The games described in this section will now be considered in terms of the theory of the fuzzy sets. The lack of operationalization has not yet allowed them to become practically used. The classical theory of games assumes that interpersonal conflict situations can be precisely described mathematically. The assumption made in this context is that the elements of a particular game can be represented as

sharply defined sets. This involves an analysis of given mathematical expressions. For more stringent requirements to modelling the existence of clearly defined sets can not be postulated. The elements of the game are affected by various sources of fuzziness. The Gain or payoff function is not always defined numerically or sharply. The strategies are employed by players are usually marked by different levels of significance and intensity. These and other conditions account for the need to include the theory of fuzzy sets in the solution concept of the theory of games. For two players employing the defined strategy sets that are wholly or partially comprised of fuzzy information of the fuzzy game  $\lambda u$  can be written as follows.

$\lambda u = \{ ( S_{1i}, u_{1i} ); ( S_{2j}, u_{2j} ); ( a_{ij}, u ) \}$   
 with  $S_{1i}$  : for  $i = \{1 \text{ to } 4\}$  strategies of player 1.  
 $u_{1i}$  : for  $i = \{1 \text{ to } 4\}$  association function for the strategies of player 1.  
 $S_{2j}$  : for  $j = \{1 \text{ to } 3\}$  strategies of player 2.  
 $u_{2j}$  : for  $i = \{1 \text{ to } 4\}; j = \{1 \text{ to } 3\}$  association function for the strategies of player 2 with respect to the strategies of player 1.  
 $a_{ij}$  : for  $i = \{1 \text{ to } 4\}; j = \{1 \text{ to } 3\}$  payoff or gain function.  
 $u_{ij}$  : for  $i = \{1 \text{ to } 4\}; j = \{1 \text{ to } 3\}$  association function for the payoff function.  
 The transition from game  $\lambda$  to game  $\lambda u$  is accomplished in three steps.

Step 1 :

A fuzzy set is defined for the set of strategies of player 1. The set of strategies and the criteria quantitatively describing the strategies are assumed to be known. An association function is calculated for each of the criteria, (ie) standard values are relativized to give the values of association. Thus, we obtain, for each strategy of player 1, a value of association for different criteria. A Set of values of association is expressed as an arithmetic mean

$$u_{1i} = \frac{1}{L} \sum_{i=1}^L u_{i1}$$

The values  $u_{ij}$  are calculated in the following matrix.

	K <sub>1</sub>	K <sub>2</sub>	K <sub>3</sub>
S <sub>11</sub>	u <sub>11</sub>	u <sub>12</sub>	u <sub>13</sub>
S <sub>12</sub>	u <sub>21</sub>	u <sub>22</sub>	u <sub>23</sub>
S <sub>13</sub>	u <sub>31</sub>	u <sub>32</sub>	u <sub>33</sub>
S <sub>14</sub>	u <sub>41</sub>	u <sub>42</sub>	u <sub>43</sub>

Step 2 :

This step is concerned with the strategies for player 2.

Fuzzy sets are defined for the strategies of player 2 and the values of association are calculated. The mapping of sets is in the form of matrix, initially signifying a basic matrix for the game to be resolved. Whereas the matrix in Step 1 was used for an additive purpose, the basic matrix is to be interpreted in terms of the games theory.

	S <sub>21</sub>	S <sub>22</sub>	S <sub>23</sub>
S <sub>11</sub>	u <sub>11</sub>	u <sub>12</sub>	u <sub>13</sub>
S <sub>12</sub>	u <sub>21</sub>	u <sub>22</sub>	u <sub>23</sub>
S <sub>13</sub>	u <sub>31</sub>	u <sub>32</sub>	u <sub>33</sub>
S <sub>14</sub>	u <sub>41</sub>	u <sub>42</sub>	u <sub>43</sub>

Step 3 :

This Step is a summary of Steps 1 and 2. This is an average of the strategy sets of players 1 and 2, with Min being chosen as a logic operator as mentioned above.

$$u_{ij} = \text{Min} ( u_{1i}, u_{1j} )$$

As a result , the fuzzy game matrix is obtained. Resolution is based on the minimax principle taken over from the classical theory of games.

	S <sub>21</sub>	S <sub>22</sub>	S <sub>23</sub>	
	-----			
S <sub>11</sub>	u <sub>11</sub>	u <sub>12</sub>	u <sub>13</sub>	
S <sub>12</sub>	u <sub>21</sub>	u <sub>22</sub>	u <sub>23</sub>	
S <sub>13</sub>	u <sub>31</sub>	u <sub>32</sub>	u <sub>33</sub>	
S <sub>14</sub>	u <sub>41</sub>	u <sub>42</sub>	u <sub>43</sub>	

#### IV. Conclusion

In this paper, The algorithm developed for fuzzy matrix games is a fuzzy concept for multi criteria decisions fuzzy matrix games multi criteria model for decision making. The concept was developed in order to take into consideration both internal and external influential variables. The result of step 1 of the fuzzy matrix game is used to assess the strategies of player 1 by values of association. This serves to express the data obtained in the strategies employed by player1. The strategies of player 2 include the use related influences. They represent what is known as external influential variables. Fuzzy matrix games provide numerous new possibilities of handling practical engineering and economic, investment planning and other problems. The solution of fuzzy matrix games constitutes a new quality of decisions representing a high degree of complexity.

#### References

- [1] Bellmann,R.E., and L.A.Zadeh. Decision making in Fuzzy environment.
- [2] Brams.S.J. Game theory and Literature.
- [3] Campos.L. Fuzzy Linear Programming Models to solve fuzzy matrix games.
- [4] Ghose.D, J.L.Speyer, J.S.Shamma. A game theoretical multiple resource interaction approach to resource allocation in an air campaign.
- [5] Friedel Peldschus, Edmundas Kazimieras Zavadskas. Fuzzy matrix games multi criteria model for Decision making in Engineering.

- [6] Maeda.T. On characterization of equilibrium strategy of two person zero sum games with fuzzy payoffs.
- [7] Peldschus.F. Research on sensitivity methods multi criteria decisions.
- [8] Peldschus.F. and E.K.Zavadskas. Matrix games in Building Technology and management.
- [9] Von Neumann. J. And O.Morgenstern. Theory of games and economic behaviour.
- [10] Vorobjov .N. Game theory in technological sciences.
- [11] Zadeh.L.A. Fuzzy Sets.
- [12] Zavadskas.E.K. Multiple criteria decision in construction.

#### AUTHOR DETAILS

(1) **R. SENTHIL KUMAR** is working as Assistant Professor in Department of Mathematics, CMS College of Science & Commerce, Chinnavedampatty, Coimbatore, Tamilnadu, India.

(2) **Dr.S.KUMARAGHURU** is working as Assistant Professor in Mathematics, PG and Research Department of Mathematics, Chikkanna Government Arts College, TirupurS, Tamilnadu, India.